

Guidelines and Fundamental Considerations for Axle Balancing



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GUIDELINES AND FUNDAMENTAL CONSIDERATIONS FOR AXLE BALANCING

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Axle assembly balancing presents a unique set of challenges. The axle assembly itself is a fairly complex device that typically has two components rotating at different speeds, gear meshes and sometimes a clutch or locking mechanism. These guidelines are set forth as an aid to understanding the essentials and as a description of some less obvious considerations important to the balancing process. As complexity increases and balance tolerances drop, it becomes increasingly important to maintain a grasp on how the fundamentals of two-plane balance measurement apply to rigid rotor balancing.

It is recommended that the reader review the section on terminology and definitions as they are essential for a proper understanding of the topics presented. The terms static unbalance and force unbalance are used interchangeably in this paper.

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The Axle Assembly

The Axle Assembly or Final Drive Unit transmits driving torque from the prop shaft to the axles. It includes an input drive flange, pinion shaft, pinion gear, ring gear and a differential unit.



The differential unit transfers torque to the right and left wheels and allows for unequal wheel speed during cornering. There are various styles of differential locking devices and clutches that limit wheel slip. These are not shown in the figure above and are not discussed in detail in this paper.

Balancing an axle assembly typically means balancing the pinion shaft as well as those components that are attached to it and turn at the same speed as the prop shaft. The rotor is normally a rigid assembly that includes the pinion gear, pinion shaft and flange. Balance corrections are almost exclusively single-plane corrections. As such, only the static unbalance can be fully corrected. This is consistent with most axle assembly balance specifications that only limit the maximum static unbalance.

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An axle assembly is normally balanced by drilling holes in the drive flange or by welding weights to the flange or slinger. Holes are normally variable depth blind holes. Some flanges have restricted zones where holes may not be drilled due to inadequate material depth or interference with other features. Weld weights may be fixed or variable length and can normally be placed at any angular location. Whether drilling or welding, it is important to evaluate maximum and minimum corrections in comparison to the range of expected incoming unbalance.

The axle assembly may have a clutch assembly that regulates torque transmission from the flange to the pinion gear. It should be recognized that such a clutch splits the pinion shaft into two separate shafts and adds a stack of clutch discs that are guided and rotate with one or the other shaft section. The clutch discs are normally loosely guided and can be a source of significant balance uncertainty. The two shafts are synchronous when the clutch is engaged but are not angularly registered to one another. It is important to understand the allocation of the balance tolerance among these components. Balance criteria may be established separately for each of these components or may be combined into a single assembly level tolerance. If the tolerance is shared with a clutch mechanism, multiple measures are required to properly characterize the combined unbalance of all components. Balance correction at the axle assembly level is normally limited to the flange and those portions of the rotor and clutch that are rigidly attached to the flange. Since only the flange is available for balance correction the clutch and the pinion gear end of the pinion shaft must be balanced prior to assembly. See appendix E for additional considerations.

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Balancing Considerations

Balance Measurement Systems

There are two different techniques commonly employed in the measurement of dynamic unbalance. These are described as soft or hard and refer to the nature of the suspension supporting the spinning part. A balancer with suspension resonance below the rotating part speed is termed soft and those with suspension resonance above are termed hard.

Soft Suspension Balancing. The oldest balancing methods involve measuring the lateral motion of a rotor as it spins in a soft suspension. This method of soft balancing directly measures the motion of the part as it is free to rotate about its central principal axis. The motion is measured at one or two planes. Depending on the sensors employed the fundamental measurement may be displacement, velocity or acceleration. Through integration or differentiation, each of these can be derived from the others. Rotor motion is dependent on the mass properties of the rotor and, to a lesser extent, the suspension. Unbalance alters the rotor's mass properties by shifting the mass center and/or principal inertia axes.

Single-Plane Soft Suspension Measurement. To simplify the acquisition of force unbalance measurements, the soft suspension can be constructed to reject couple unbalance. The suspension can be made to be stiff in torsion yet soft in translation by employing a leaf spring or hinge to limit couple (out of phase) motion. In this case, the rotor is not free to rotate about the central principle axis but is restricted to an axis parallel to the axis of rotation. This displacement can be measured with a single pickup and calibrated with a single influence coefficient vector to give force unbalance. Suspensions of this type are able to reject up to 75-95% of the couple motion.

Two-Plane Soft Suspension Measurement. A two-plane suspension allows two degrees of freedom for the spinning rotor. Side to side lateral displacements of the central principal axis will either be in phase or out of phase at the sensors. These motions are related to static and couple unbalance respectively. Unbalance can be determined from the motion either analytically using the known mass properties of the rotor and suspension or by employing a calibration process. Both approaches provide a solution that is unique to any single part configuration. Calibration is normally performed using trial weights in a process similar to the one described in Appendix A. For a soft suspension balancer, calibration only holds for parts with identical mass properties *i.e.* same weight and inertia properties. The calibrated measurement system can provide unbalance in two planes in units of force unbalance or weight at radius.





Hard Suspension Balancing. The hard suspension or hard bearing balancing system is generally more versatile than the soft bearing system. Pioneered by BTI and others, this method measures the forces generated by unbalance as the rotor spins on its axis of rotation. These forces are unaffected by the fundamental mass properties of the rotor. This is a great advantage when balancing rotors with significant part to part weight variation and it even allows balancing a wide variety of parts with different mass properties without re-calibration. Hard bearing systems are often permanently calibrated to traceable standards and read out unbalance directly in units of force unbalance.

Hard bearing systems consist of a rigid base that supports a measuring platform through two force measuring elements. The force measuring elements are usually ceramic or quartz piezoelectric elements. The platform suspension will normally have other support features to restrict motion to a single-plane. The stiffness of these supports should be subordinate to that of the sensing elements so that the largest percentage of unbalance force passes through the two force sensors. The actual displacement of the platform is very small since it is only due to the compression of the piezoelectric elements. Displacements are often only a few nanometers resulting in adequate suspension stiffness over the intended running speed.

The resonance requirement leads to rigid structures that must resist the unbalance forces with minimum flex. Hard bearing balancers have evolved to be very heavy duty, robust machines. The balance platform itself must also be stiff and lightweight and is ideally suited for complex work holding solutions. Axle assemblies often require unusual work holding that can be readily adapted to the measurement platform.

Hard bearing balancers normally employ two sensing elements and as such are two-plane balance measuring systems. As noted before, soft suspension machines may have a suspension that mechanically rejects couple motion. This is not necessary with hard suspension machines. It is difficult to design a suspension sufficiently stiff in rotation to get complete couple rejection. Without a means to mechanically filter the forces and/or motion associated with couple unbalance, any single sensor measuring system runs the risk of recording the response at a nodal point. The relationship between force unbalance and sensor output can be non-linear and unreliable near such a node. On the other hand, couple rejection is easily performed mathematically from the two-plane data. A properly calibrated two-plane system can reject more than 99% of the couple unbalance. Isolating force unbalance to a degree of accuracy much greater than what could be accomplished mechanically.

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Rotor Components and Unbalance

Balancing an axle assembly typically means balancing the pinion shaft and those components that are attached to it. The most common configuration is a one piece pinion shaft and gear with a drive flange.



When assembled, the pinion shaft and drive flange form a simple rigid rotor. The total unbalance of the rotor includes static and couple unbalance in the pinion shaft, the flange itself and the unbalances associated with mounting the drive flange and prop shaft.

The one piece pinion gear and shaft is normally almost fully machined and has relatively small residual unbalance. The axis of rotation is set by the pinion bearings. They are piloted on the pinion shaft and if the rotor was only composed of the pinion shaft, the rotor would be well balanced. However, the drive flange must be added. The drive flange itself is often balanced as a component and will also have relatively small residual unbalance. While the components may be well balanced, imperfections in the assembly and alignment of these two balanced components lead to an unbalanced rotor.

Bending of the pinion shaft has also been identified as a source of unbalance in cases where the bearing preload technique does not provide for a well centered axial load. Pinion shaft bending has the potential to unbalance the pinion shaft itself as well as exaggerate mislocation of the flange.

The flange is normally fitted to the shaft with a spline so that engine torque can be transmitted to the axle. The flange's position is established by eccentricity of the spline as well as the squareness of the shoulder on which it rests. These geometric errors give rise to two significant sources of unbalance.

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1) Misalignment of the flange with respect to the axis of rotation.



2) Misalignment of the prop shaft and its associated end weight with respect to the axis of rotation.



The unbalance associated with both the flange and the prop shaft end fitting is equal to a weight times an eccentricity (ignoring couple unbalance due to rotation of inertia axes). The effects of both can be combined and expressed as the total weight times an eccentricity.

For the most part, axle assembly balancing is a correction of these two components. End fitting unbalance is addressed at some length in following sections. While the end fitting can be a significant source of unbalance, it may or may not be included in the axle assembly balance tolerance. Whether it is or not, it must be recognized that any prop shaft or drive mechanism that attaches a weight to the flange will be contributing to the system unbalance.

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Balance Specifications

The three primary goals of balancing are:

- to improve the performance,
- to improve the reliability or service life,
- to improve the end customer's perception of quality.

A balance specification sets a limit for the amount of residual unbalance on a part or assembly in order to achieve a combination of these goals. The third reason is a prime motivation for axle balancing *i.e.*, reducing the vibration perceived by the final customer. Since the goal is to reduce perceived vibration the balance specification should be targeted to this goal.

Three choices in specifying the maximum allowed unbalance are: 1) specify a force unbalance, 2) specify force and couple unbalance and 3) specify two-plane unbalance with plane locations

1) Specifying Force Unbalance Only. This is the most common form of specification for an axle assembly, rooted in the availability of only one correction plane and an insensitivity to couple vibration. The vehicle normally has a low sensitivity to couple induced vibration and the pinion shaft has an inherently low couple unbalance, so it is reasonable to ignore couple unbalance altogether. In these cases a limitation on static unbalance is sufficient to satisfy the objective and is consistent with the single correction plane. It is particularly effective with short pinion shafts whose unbalance is largely due to miss-location of the drive flange as described in the following section on Drive Flange Unbalance. It is less effective for longer pinion shafts or those that have significant unbalance associated with the pinion end.

2) Specifying Force and Couple Unbalance. Limits on both force and couple unbalance completely specify the allowed unbalance in any rigid rotor. In this case the couple specification has units of $[M\cdot L^2]$ and requires a location for the mass center to complete the specification. For the purpose of establishing correlation between unbalance and the perception of vibration, both force and couple unbalance levels should be considered. This method is generally the most useful expression of unbalance and can be used during the design stage to reduce both force and couple unbalance to acceptable levels. Subsequent manufacturing variations generally re-introduce unbalance, making balance correction necessary.

As in any case that involves a two-plane balance specification and only a single correction plane, the acceptable residual unbalance will be some combination of force and couple. Except for special cases, it is not possible to reduce both force and couple unbalance to zero with the single correction. Some special cases are discussed in the section on Single-Plane Correction. The relationship between a single-plane correction at the flange and a two-plane specification may by non-intuitive; however, it is mathematically straightforward to calculate the effects of a single-plane correction on both force and couple unbalance levels. It is always possible to reduce the static unbalance to zero

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with a single-plane correction. However, the ability to change couple is limited. A single-plane correction only allows couple correction to the extent allowed by the distance between the correction plane and the rotor mass center. The couple specification can be set at a high level to serve as a guard against serious manufacturing defects. Parts with high couple unbalance would be rejected as uncorrectable and scrapped or re-built.

3) Specifying Two-Plane Unbalance. A two-plane specification is written to allow a maximum residual force unbalance in each of two specified planes. This is often referred to as a right-left correction. With knowledge of mass center location, a right-left expression of unbalance can be translated into force-couple unbalance. Plane locations play a significant role. Planes may be chosen at key attach points, bearing locations or more commonly, correction points. If the right plane is chosen as the flange correction plane, the left plane location is somewhat arbitrary but can be used to influence the residual unbalance after applying the right plane correction. This is discussed at some length in the section on Single-Plane Balance Corrections. Tolerance planes can be established at other key locations to more directly evaluate and limit the unbalance forces at these locations. This is not uncommon for motor and fan applications but has not been seen in an axle assembly.

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Drive Flange Unbalance

A compound rotor that is constructed from two well-balanced components will often exhibit a state of quasi-static unbalance. The same state of unbalance will occur if a balanced rotor is unbalanced with a single weight placed some distance away from the composite mass center.



For a well-balanced pinion shaft with a well-balanced flange, mounting eccentricity will produce a quasi-static state of unbalance.



The combination of static and couple unbalance can be corrected with a single balance correction if the correction is made at the proper location. While the rotor has both static and couple unbalance, the entire unbalance is attributed to eccentricity of a single weight or mass. If the couple unbalance created by flange tilt is ignored, the best correction location is a plane that contains the mass center of the flange. Correction in any other plane will leave additional couple unbalance.

This leads to a discussion of the correction technique and a closer look at single- plane and twoplane balance correction.

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Single-Plane Balance Correction

Single-plane balance correction works best when applied to rotors that are disk like. Unbalance in a disk shaped rotor can often be almost entirely attributed to eccentricity of the rotor itself with respect to the axis of rotation. A balance correction made on the disk is often made very close to the plane that contains the mass center and consequently is very effective in drawing the mass center back to the axis of rotation without introducing significant couple. There is little need for a second correction plane as the levels of couple unbalance are low. Rotors that are not disk-like are prone to higher levels of couple unbalance. It is more likely that correction planes on longer, cylindrical rotors will be some distance from the rotor mass center and each correction will naturally and simultaneously alter both the static and couple unbalance.

In the case of the pinion shaft, only one correction plane is allowed for a rotor that is more like a cylinder than a disk. The location of the balance correction plane is such that its influence on static and couple unbalance cannot be ignored. For this discussion the right plane is the correction plane and is located on the flange. Correction weight will be added or removed in the right plane. When referring to the flange, reference is being made to the combined weight and unbalance of the flange with an attached end weight. Since there is only one correction plane, the location of the other plane, the left plane, is somewhat arbitrary. Furthermore there is the dilemma of which correction is the best correction: 1) a force correction, 2) a right plane correction or 3) some combination of force and couple corrections.

1) **Force Correction.** It is easily shown that the force unbalance correction is independent of the left plane location. Placing the entire force correction in the right plane (or any plane) will reduce the static unbalance to zero. The couple unbalance will, in general, be altered but not necessarily reduced. If the axle assembly rotor fits the description offered earlier of quasi-static unbalance, it is likely that the residual couple will be substantially reduced since the right plane and the flange mass center are generally very close together. In this case, the amount of residual couple [M·L2] will be equal to the force correction times the distance between the correction plane and the flange mass center. In the more general case, the residual couple will be the total of the original couple plus the force correction times the distance between the correction plane and the midpoint between the correction planes. The resulting couple may be more or less than the original couple depending on the sign of the correction with respect to the initial couple.

2) **Right Plane Correction**. By definition a right plane correction should be accompanied by a left plane correction. Together the combination of right and left plane corrections can eliminate static and couple unbalance in a rigid rotor. In the case where only the right plane correction is made, residual unbalance will depend on the nature of the initial unbalance and the choice of left plane location. Three cases are presented: a) left plane location, b) unbalance in right plane and c) unbalance near right plane.

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a) Left Plane Location. Consider a rotor with left and right corrections (unbalance) already defined.



Equivalent force systems are constructed such that a summation of forces and moments about any point have the same resultants for either case. Moving the left correction further to the left, an equivalent correction can be constructed:



Where M is a couple unbalance in $[M \cdot L^2]$. The couple unbalance can be expressed as a force couple with vectors in the left and right planes.

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By defining n as the ratio of D/d and expressing the moment as a two force couple, an equivalent free body diagram is drawn.



The couple correction is out of phase with the original left plane correction and in phase with the original right plane correction. The magnitude of the couple correction must equal the resultant moment

$$M = C \cdot (n \cdot d)$$

or

$$C = M/(n \cdot d),$$

developing the expression for moment

$$M = L \cdot (D - d) = L \cdot (n \cdot d - d) = L \cdot d \cdot (n - 1)$$

and

$$C = [(n-1)/n] \cdot L.$$

For large values of n e.g., the left correction to the extreme left, the magnitude of the couple correction approaches that of the original left plane correction. The new left plane correction approaches zero while the new right plane correction approaches R+L. This is nearly equivalent to making the entire force correction in the right plane.

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b) Unbalance in Right Plane. Consider the case where the unbalance is located in the right plane.



It is assumed here that the unbalance weight is small with respect to the rotor weight. Unless the mass center lies in the right plane, such a rotor will exhibit a quasi-static state of unbalance. If the mass center lies in the right plane, couple unbalance will be zero and the rotor is a case of simple static unbalance. The static unbalance is shared between right and left planes in inverse proportion to their distance from the mass center.

$$L_{Static} = m \cdot r \cdot \frac{a}{D}$$
$$R_{Static} = m \cdot r \cdot \frac{(D-a)}{D}$$

Couple unbalance is represented equally in both planes with opposite signs.

$$L_{Couple} = -m \cdot r \cdot \frac{a}{D}$$
$$R_{Couple} = m \cdot r \cdot \frac{a}{D}$$

Combining the components of left and right unbalance

$$L = 0$$
$$R = m \cdot r$$

The right plane correction contains the entire balance correction and is entirely independent of both its location with respect to the mass center and the distance between planes.

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c) Unbalance Near Right Plane. Consider the more general case where the unbalance is located near the right plane.



Again it is assumed here that the unbalance weight is small with respect to the rotor weight.

$$L_{Static} = m \cdot r \cdot \frac{c}{D}$$
$$R_{Static} = m \cdot r \cdot \frac{(D - c)}{D}$$

Couple unbalance is the same as before.

$$L_{Couple} = -m \cdot r \cdot \frac{a}{D}$$
$$R_{Couple} = m \cdot r \cdot \frac{a}{D}$$

Combining the components of left and right unbalance

$$L = m \cdot r \cdot \left(\frac{c-a}{D}\right)$$
$$R = m \cdot r \cdot \left(\frac{D-c+a}{D}\right)$$

Using a + b = c, the left and right expressions become

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$$L = m \cdot r \cdot \left(\frac{b}{D}\right)$$
$$R = m \cdot r \cdot \left(\frac{D - b}{D}\right)$$

The left plane correction represents the residual unbalance after making the right plane correction. Note that the magnitude of the residual unbalance depends on the ratio of *b* to *D*. For b = 0 or for *D* much larger than *b*, the residual unbalance is negligible and both expressions reduce to those presented in the previous case where the unbalance weight is in the right plane. Otherwise for $b \neq 0$, the left plane correction is non-zero and is proportional to

the ratio of
$$\frac{b}{D}$$
.

3) Force and Couple Combination. Arguments can be made for constructing balance corrections that include pre-determined ratios of force and couple unbalance. This is application specific and is not discussed at any length here in favor of the more general cases already presented. However, it is recommended that the interested reader evaluate residual unbalance (force and/or couple) for various correction ratios and various plane locations. It is possible that the residual unbalance can be favorably biased by the proportions of force and couple in the balance correction. This approach can be extended to include optimization methods to minimize a combination of force and couple unbalance.

A right plane correction is equivalent to a force correction when the unbalance is caused by a single weight in or near the right plane and the left plane is chosen sufficiently far to the left. Case 2c can be used to evaluate any combination of force and couple provided the corrections all lay on the same line (quasi-static unbalance). For the more general case where force and couple unbalance are not aligned, it becomes a choice or preference for the nature of the residual unbalance. For longer shafts that have unbalanced components on the left side, a force correction in the right plane may leave a large couple unbalance due to the distance between right plane and the mass center. On the other hand, a true right plane correction will leave minimum residual unbalance in the right plane with little or no effect on the left plane unbalance. See Appendix F for sample data and a comparison of single-plane corrections.

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End Weight Unbalance

Geometric errors in the location and orientation of the pinion flange datums turn into unbalance with mounting of the prop shaft to these datums. It is possible to calculate this unbalance by determining how far off center the prop shaft end weight is placed as a result of these geometric flaws. This analysis is useful in evaluating repeatability as well as unbalance bias.

It is first necessary to know the mass properties of the end weight. If the end weight is not the prop shaft itself, it should be representative of the prop shaft. A balance bias will exist between measurements with tooling of different weights.



A portion of the prop shaft weight acts at the pivot point. This weight is designated n-W2 where W2 is the total weight of the prop shaft itself (less end fittings). There are two geometric errors associated with the pinion flange that create unbalance: 1) eccentricity of the pilot diameter and 2) non-perpendicularity of the flange.

1) **Pilot Eccentricity.** This is eccentricity with respect to the pinion shaft centerline of rotation.

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The magnitude of eccentricity is denoted δ_0 . It is truly a vector quantity with angle and magnitude. Bold face variables are used to denote those quantities that should be treated as vectors. The unbalance associated with eccentricity is

$$U_{Ecc} = (W1 + n \cdot W2) \cdot \delta_0$$

where

$$\begin{split} &W1 = end \ fitting \ weight \\ &n \cdot W2 = shaft \ end \ weight \ (concentrated \ at \ pivot) \\ &\delta_0 = centering \ error, \ a \ vector. \end{split}$$

The variable n represents the percentage of the total weight of the prop shaft, W2, associated with the end that is attached to the flange.

2) **Non-Perpendicularity.** This is a case where the pinion flange face is not exactly perpendicular with the centerline of rotation.



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The magnitude of non-perpendicularity is denoted by the angle, θ . It also is a vector quantity and should be expressed with angle and magnitude, $\theta \cdot \boldsymbol{\theta}_{\theta}$, where $\boldsymbol{\theta}_{\theta}$ is a unit vector indicating the angular orientation of the offset. This provides a means to express the direction as well as the magnitude of non-perpendicularity. It is more convenient to calculate and use the eccentricity vectors associated with this angle vector. There are two vectors, $\boldsymbol{\delta}_1$ and $\boldsymbol{\delta}_2$. One is associated with the mass center and the other with the pivot point. The unbalance associated with non-perpendicularity is then

$$U_{Perp} = W1 \cdot \boldsymbol{\delta}_1 + n \cdot W2 \cdot \boldsymbol{\delta}_2$$

where

$$\begin{split} & \boldsymbol{\delta}_1 = X1 \cdot sin \boldsymbol{\theta} \cdot \boldsymbol{\delta}_{\boldsymbol{\theta}} \\ & \boldsymbol{\delta}_2 = X2 \cdot sin \boldsymbol{\theta} \cdot \boldsymbol{\delta}_{\boldsymbol{\theta}} \\ & \boldsymbol{\delta}_{\boldsymbol{\theta}} \text{ is a unit vector indicating the direction of offset} \end{split}$$

All together the unbalance associated with the geometric errors is

 $\mathbf{U}_{\text{Total}} = \mathbf{U}_{\text{Ecc}} + \mathbf{U}_{\text{Perp}}$

Repeatability can be assessed by introducing the expected variation in offset or angle and calculating the related unbalance.

For the same mounting errors, a balance correction error or bias is introduced when tooling used to measure unbalance does not match the production component weights and geometry.

Assume that the tooling and the production components center and square on the same features:

δ₀ = centering error θ = squaring error.

The balance error introduced is

The individual quantities are calculated as outlined above. Again, these are all vector quantities.

See Appendix C for a method of combining W1 and W2 into a single equivalent end weight.

See Appendix D for further description of the prop shaft end weight.

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Effect of Residual Unbalance

Bearing Reaction Forces

A one-piece pinion shaft is normally supported by two preloaded, tapered roller bearings. Even at 5000 rpm, forces due to unbalance will only be a few pounds and nearly insignificant compared to the pinion gear contact forces. Unbalance at or below typical tolerance levels is not a factor in bearing wear and life.

Vehicle Vibration

Vehicle sensitivity to static and couple unbalance can be evaluated analytically or empirically. Both can be very costly and complex. It is far beyond the scope of this paper to address either method in detail, however it is useful to point out how balancing data can support evaluation of vehicle vibration.

For a dynamic analysis, right and left plane unbalance values can be translated into equivalent unbalance in any other two planes. This technique can be used to estimate forcing functions at support points by translating unbalance to the points of interest. Axle assembly mount stiffness should be considered in the analysis as a resilient mount may provide some degree of isolation in the normal speed range. Some difficulty may be encountered distributing the unbalance in cases where the axle assembly is over-constrained – where there are multiple paths for the force and/or couple unbalance forces to be reacted. Assumptions and results should be validated in vehicle tests.

More subjective, empirical testing is common for NVH evaluations. Various levels of force and couple unbalance can be applied to an axle assembly which is then evaluated in a test vehicle. Driver evaluations are compiled to establish limits for vibration and unbalance. This technique is widely used for HVAC and drive train components.

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Terminology and Definitions

Balancing terminology used in this paper is consistent with that used in ISO 1925, Balancing Vocabulary. A few of the more prominent terms are defined below.

AXIS OF ROTATION: The instantaneous line about which a body rotates. In the case of a rigid rotor supported by rigid bearings, the axis of rotation is the shaft axis.

CENTRAL PRINCIPAL AXIS: The principal inertia axis that is most nearly coincident with the shaft axis. The mass center lies on the central principal axis. This is also known as the balance axis or the mass axis.

COUPLE UNBALANCE: The condition of unbalance for which the central principal axis intersects the shaft axis at the mass center. By definition, the central principal axis and the shaft axis are not parallel. The units of couple unbalance are $[M \cdot L^2]$ or $[W \cdot L^2]$.



DYNAMIC UNBALANCE: The condition of unbalance in which the central principal axis has any position relative to the shaft axis. This is the most general case of unbalance. It includes static and couple unbalance as well as the combined case in which the central principal axis is not parallel to and does not intersect the axis of rotation.



Since dynamic unbalance may be a combination of static and couple unbalance and static and couple unbalance have different units, there are no unique units for dynamic unbalance. It can be expressed as static and couple or in terms of the balance corrections required.

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QUASI-STATIC UNBALANCE: The condition of unbalance for which the central principal axis intersects the shaft axis at a point other than the mass center. This is a special form of dynamic unbalance in which the static and couple unbalance force vectors lie in the same plane. As with dynamic unbalance, there are no unique units for quasi-static unbalance.

SHAFT AXIS: The straight line joining the journal centers. In the case of a rigid rotor supported by rigid bearings, the shaft axis is the axis of rotation.

STATIC UNBALANCE: The condition of unbalance for which the central principal axis is only displaced parallel to the shaft axis. Static unbalance is also known as Force Unbalance. The units of static unbalance are [M·L] or [W·L].





Appendix A

Two-plane Calibration

The calibration technique employed by BTI is an application of the method for two-plane influence coefficient balancing developed by Thearle (1934). The balance platform has two sensors that give the balancer the ability to measure static unbalance, couple unbalance or any combination of the two. The influence coefficient method is well documented and only a brief description is offered here. First, two calibration planes are defined and calibration weights are selected.



A and **B** are the unbalance signals and **L** and **R** will be known unbalances. By placing weights in one or the other calibration planes, a known unbalance is applied. Three measurement spins are required: weight in the left plane, weight in the right plane and no weights. The measured signals and known unbalances are all vector quantities that can be expressed as amount and angle or, for rotation around the Z axis, as vectors in the X-Y plane

 $\mathbf{A} = A_x \cdot \mathbf{i} + A_y \cdot \mathbf{j}$ $\mathbf{B} = B_x \cdot \mathbf{i} + B_y \cdot \mathbf{j}$ $\mathbf{L} = L_x \cdot \mathbf{i} + L_y \cdot \mathbf{j}$ $\mathbf{R} = R_x \cdot \mathbf{i} + R_y \cdot \mathbf{j}$

Here **i** and **j** correspond to unit vectors along the X and Y axes respectively. The X and Y components correspond in turn to the cosine and sine components of the dynamic signal.

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The balancer mathematically sorts out the known unbalance and populates two influence coefficient matrices that relate the known unbalance to the measured signals.

$$\begin{cases} L_{X} \\ R_{X} \end{cases} = [\alpha] \cdot \begin{cases} A_{X} \\ B_{X} \end{cases}$$
$$\begin{cases} L_{Y} \\ R_{Y} \end{cases} = [\beta] \cdot \begin{cases} A_{Y} \\ B_{Y} \end{cases}$$

 $[\alpha]$ and $[\beta]$ are 2 x 2 matrices of influence coefficients. For subsequent balance measurements the measured signals, **A** and **B**, can be converted into unbalance in the left and right calibration planes, **L** and **R**. Unbalance in these two planes can be translated into any other two planes, e.g. correction planes and tolerance planes. The unbalance can also be expressed as force and couple unbalance with respect to any single-plane or point on the axis of rotation.

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Appendix B

Two-plane Balance Correction

A two-plane balance measurement captures enough information to calculate corrections for both static and couple unbalance. These corrections are displayed as left and right plane corrections. A portion of each correction will serve to correct either static or couple unbalance. The proportion of each depends on the point of reference; i.e., the point at which one intends to make the entire force correction. It is not necessary to know where the mass center of the rotor lies along the axis of rotation to calculate the proper left and right corrections. However, by moving the corrections to the mass center and calculating an equivalent force and couple system, the true unbalance of the rotor can be determined. The convention employed by BTI arbitrarily defines force correction as the portion of the left and right corrections that are in phase and couple correction as the portion that is out of phase. This is equivalent to locating the point of reference midway between the left and right corrections

L = left plane correction R = right plane correction

The equivalent force and couple corrections, F and C, are

$$F = L + R$$
$$C = \frac{1}{2} \cdot (L - R)$$

To perform a force-couple correction the entire force correction, **F**, is made at the midpoint and the couple correction, **C**, is made in the left and right planes. The left plane couple correction is at the angle specified and the right plane couple correction at an angle 180° out of phase with that specified. Note that the vector **C** is used here to indicate a couple correction and will have units of [M·L]. It is related to the couple unbalance, U_c with units of [M·L²], by the distance between the corrections. If d is the normal vector from the left plane to the right plane then

$$U_c = L x d$$

This is a cross product and results in a vector that lies normal to the couple correction vectors, **C**. It properly reflects the axis about which the couple moment acts.

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The couple corrections can actually be made in any pair of planes that have the same separation distance as the right and left planes. Or more generally, can be made in any pair of planes with an adjustment for the distance between planes. If **L** and **R** are calculated for a distance between planes of d, then the adjusted corrections, **L'** and **R'**, for a different distance, d', are

$$\mathbf{L}^{\prime} = (d/d') \cdot \mathbf{L}$$
$$\mathbf{R}^{\prime} = (d/d') \cdot \mathbf{R}$$

Corrections in these two planes can be translated into any other two planes using simple static analysis and constructing an equivalent force system. The corrections can also be expressed as force and couple unbalance with respect to any single-plane or point on the axis of rotation using the same technique. The following are all equivalent force systems and illustrate moving a force correction from point O to point P.



The force correction is equal to the true static unbalance in the rotor and as such can be made in any plane. The couple correction will vary depending on where the force correction is made. If the force correction is made in the plane that contains the rotor's mass center, the couple correction will reflect the true couple unbalance in the part and is related to the tilt of the inertia axes.

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Appendix C

Equivalent End Weight

An end weight has two discrete weights that are of interest in the balancing application. It is often useful to combine these two discrete weights into one equivalent weight. This is a mass center problem. As before, the end weight can be described in terms of weights W1 and n·W2 at distances X1 and X2 from the mounting face.



Or a more generic representation:



The equivalent end weight will have weight W at a distance L from the mounting face as shown on the right.

For a linear offset, the location of the end weight is unimportant. However it should be clear that the total weight must match the sum of the two discrete weights

$$W = W1 + n \cdot W2$$

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The angular offset is used to determine the proper location for the mass center.



The unbalance associated with an angular offset is

 $\begin{array}{ll} U_{Total} &= W1 \cdot \delta_1 + n \cdot W2 \cdot \delta_2 \\ &= W1 \cdot X1 \cdot \sin \theta + n \cdot W2 \cdot X2 \cdot \sin \theta \\ &= (W1 \cdot X1 + n \cdot W2 \cdot X2) \cdot \sin \theta. \end{array}$

If the unbalance of the single weight is to be equivalent to the total unbalance of the two, then

 $W \cdot L \cdot \sin\theta = (W1 \cdot X1 + n \cdot W2 \cdot X2) \cdot \sin\theta$ $W \cdot L = (W1 \cdot X1 + n \cdot W2 \cdot X2)$

and

 $L = (W1 \cdot X1 + n \cdot W2 \cdot X2)/(W1 + n \cdot W2).$

If X1 = X2, this simplifies to L = X1 = X2 and is only significant when X1 \neq X2. This is often the case when CV joints or rubber couplings are used to join the prop shaft to a pinion flange. The influence of weight differences should not be overlooked.

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Appendix D

Prop Shaft End Weight Calculation

A fully assembled prop shaft generally has 3 components joined at 2 pivot points. The prop shaft is joined to the pinion flange and drives the differential through the pinion flange.



The total weight of the prop shaft is the sum of these 3 individual components,

$$W_{TOTAL} = W1 + W2 + W3.$$

The 2 components on the left hand side, the end fitting and the prop shaft, can have a significant contribution to the system unbalance when mounted to the pinion flange on the differential. The mounting is seldom perfect and will usually have radial runout and face runout. These runouts may result in a centerline offset, δ , of the pilot diameter and a flange mounting face that deviates from perpendicular by a small angle, θ . Both are measured with respect to the centerline of rotation of the pinion shaft.



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These errors may be independent of each other. They position the mass center off of the centerline of rotation of the pinion shaft – resulting in unbalance.

If the mass properties of the prop shaft are understood, it is possible to include a balance correction to the pinion and reduce the system unbalance. To calculate the correction it is necessary to know the weights of the prop shaft and the pinion side end fitting. The axial locations of the respective mass centers is also required, along with the length of the prop shaft and the location of the pinion side pivot point.



If the prop shaft is symmetric or nearly symmetric, there is little error in assuming that the mass center is at the midpoint between the pivots. Each pivot point will then support half the weight of the prop shaft.

If the difference between X1 and X2 is small or if X1 and W1 are small with respect to X2 and n·W2, the end weights can be estimated by measuring the end weights of the prop shaft when simply supported at the pivot points. Only the weight of the pinion end is of interest.



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Appendix E

Balancing Axles with Clutches

Balancing an axle assembly with an electronic controlled torque transfer device requires special consideration. A variety of torque transfer control devices exist. In some designs an electronic clutch is inserted between the input flange and the pinion shaft. This effectively splits the pinion shaft into two separate rotors with random phase angle to one another. Only the flange side of this combination is available for balance correction. Three related areas are considered: 1) the balance measurement process, 2) balance variation of the clutch and 3) clutch bearing design requirements for balance.

1) **Balance Measurement Process.** For an axle assembly with a clutch, the balance process is commonly split into two individual balance measurements with a "clocking" process in between the two measurements. This adds the requirement that the balancer be able to command a torque and hold the output shafts during the clocking. Balance measurements are taken with the clutch at full torque so that the pinion and flange do not slip with respect to each other. Between the two measurements the holding torque is reduced and the input is turned 180 degrees while the output is held. The second measurement is then taken at full torque and the flange unbalance data is derived as the vector average of the two readings. The clutch unbalance is derived (for reference) as half the vector difference of the two readings.

It is also common to have a "pre-clock" step before the first measurement to set the clutch discs in position. The clutch is composed of many friction discs that are loosely guided on either the flange end or the pinion end of the shaft. The discs have some freedom to move laterally and the ability to alter the unbalance of both ends of the shaft. A pre-clock step often improves repeatability. This is accomplished by holding the output, applying partial torque, and rotating the input one or more turns.

2) **Balance Variation from the Clutch**. Even with a pre-clocking step, the balance variations from the clutch may be high with respect to tolerances. Any lateral motion of the clutch discs or other loose clutch components can be a source of significant balance uncertainty. Appropriate tolerance specifications are required as are adequate quality management practices.

The addition of a joint, typically a spline joint, between the clutch and the pinion shaft introduces another source of unbalance and unbalance variation. Various bearing configurations lead to the pinion end centering the flange end or vice versa. This interface needs to be understood with regard to axle assembly unbalance. Runout in this joint leads to unbalance that may be associated with the pinion end of the shaft which is concealed within the clutch assembly and inaccessible for balance correction after assembly.

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3) **Clutch Bearing Design Requirements**. A third consideration is the accuracy of the bearing supporting the flange. Since this bearing is now part of the clutch it does not have the function of guiding the pinion gear, therefore the requirements on clearance and stiffness are often relaxed significantly. This can be in conflict with the balance tolerances, especially when the suspended mass of the driveshaft is considered. This must be recognized in the design process so that the clutch bearings can be specified to meet clearance and runout specifications consistent with the balance requirements.

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Appendix F

Sample Data and Single-Plane Correction Comparison

The following examples are drawn from actual test data. Two parts are compared; sample part 2 has the higher levels of initial unbalance. The axle assembly is generic in nature, with a typically short pinion shaft and no clutch or slip limiting devices. The right plane is the correction plane and is on the flange. Three correction techniques are evaluated: 1) force correction, 2) right plane correction with left plane at the axle centerline and 3) right plane correction with left plane 125 mm to the left of the axle centerline.



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The data is representative of most axle assemblies. The initial unbalances can be expressed in three different formats:

SAMPLE PART 1				SAMPLE PART 2				
FORCE		COUPLE		FORCE		COUPLE		
AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	
77.0 g·mm	62°	10,558.3 g·mm ²	173°	130.5 g·mm	178°	17,824.7 g·mm²	274°	
77.0 g·mm	62°	42.2 g⋅mm	263 °	130.5 g·mm	178°	71.3 g·mm	4°	
LEFT		RIGHT		LEFT		RIGHT		
AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	
15.2 g·mm	328°	79.4 g·mm	73 °	9.4 g·mm	51°	136.4 g·mm	181°	

Note that force unbalance is the same for both expressions of force unbalance but differs for couple unbalance. The difference is the plane separation or couple arm. By dividing the couple unbalance moment $[g \cdot mm^2]$ by the distance between planes (250 mm) the couple force or unbalance is calculated. Note that there is also an angle difference in the couple expressions. When expressed in $[g \cdot mm^2]$, the angle is to the axis about which the moment acts. When expressed in $[g \cdot mm]$, the angle corresponds to a couple force or unbalance in the left plane. The left plane couple force is accompanied by an equal and opposite force in the right plane. These couple forces both act in a direction normal to the moment vector.

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1) Applying the entire force correction in the right plane, the corrections and residual unbalances are:

SAMPLE PART 1				SAMPLE PART 2					
LEFT CORRECTION		RIGHT CORRECTION		LEFT CORRECTION		RIGHT CORRECTION			
AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE		
0.0 g·mm	0°	77.0 g⋅mm	242°	0.0 g·mm	0 °	130.5 g·mm	358°		
FORCE		COUPLE		FORCE		COUPLE			
AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE		
0.0 g·mm	0°	3,790.9 g·mm ²	238°	0.0 g·mm	0 °	2,339.3 g·mm²	321°		
0.0 g·mm	0°	15.2 g·mm	328°	0.0 g·mm	0°	9.4 g·mm	51°		
LEFT	T RIGHT		LEFT		RIGHT				
AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE		
15.2 g·mm	328°	15.2 g·mm	148°	9.4 g⋅mm	51°	9.4 g⋅mm	231°		

Note that force unbalance is reduced to zero and that the couple unbalance is also reduced. The fact that the couple unbalance is substantially reduced is an indication that the unbalance is largely due to a single unbalance located near the right correction plane. The small initial unbalance in the left plane is unchanged. The amount of this residual determines the final couple unbalance. Also note that left and right unbalances are equal and opposite. This reflects the fact that only couple unbalance has been left in the part.

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2) Applying the right plane correction properly in the right plane, the corrections and residual unbalances are:

SAMPLE PART 1				SAMPLE PART 2					
LEFT CORRECTION		RIGHT CORRECTION		LEFT CORRECTION		RIGHT CORRECTION			
AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	<u>AMOUNT</u>	ANGLE		
0.0 g⋅mm	0°	79.4 g·mm	253°	0.0 g·mm	0°	136.4 g·mm	1°		
FORCE		COUPLE		FORCE		COUPLE			
AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE		
15.2 g·mm	328°	1,895.4 g·mm ²	238°	9.1 g·mm	51°	1,169.7 g·mm²	321°		
15.2 _{g·mm}	328°	7.6 g·mm	328°	9.4 _{g·mm}	51°	4.7 g·mm	51°		
LEFT RIGHT		LEFT		RIGHT					
AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE		
15.2 g·mm	328°	0.0 g⋅mm	0°	9.4 _{g·mm}	51°	0.0 g·mm	0°		

Right plane unbalance is reduced to zero as it should be for a perfect correction. The residual unbalance in the left plane is the same as that left after the force correction in the previous example. Residual unbalance is a combination of force and couple unbalance.

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By moving the left plane further to the left of the axle centerline, it coincides with one of the vehicle mounts.



The initial unbalances again:

SAMPLE PART 1

SAMPLE PART 2

FORCE		COUPLE		FORCE		COUPLE	
AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE
77.0 g·mm	62°	10,558.3 g·mm ²	173°	130.5 g·mm	178°	17,824.7 g·mm²	274°
77.0 g·mm	62°	28.2 g·mm	263°	130.5 g·mm	178°	47.5 g·mm	4°
LEFT		RIGHT		LEFT		RIGHT	
AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE
10.1 g·mm	328°	78.3 g·mm	69°	6.2 g⋅mm	51°	134.4 g·mm	180°

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Note that initial unbalance is not dependent on plane location when expressed as force and couple unbalance with couple in [g·mm²]. The change in couple when expressed in [g·mm] is due to the larger distance between correction planes. Since the distance between couple unbalance forces is increased, the couple unbalance decreases accordingly.

SAMPLE PART 1				SAMPLE PART 2					
LEFT CORRECTION		RIGHT CORRECTION		LEFT CORRECTION		RIGHT CORRECTION			
AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE		
0.0 g·mm	0°	78.3 g·mm	249°	0.0 g·mm	0°	134.4 _{g·mm}	0°		
FORCE	FORCE		COUPLE		FORCE		COUPLE		
AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE		
10.1 g·mm	328°	1,263.6 g·mm ²	238°	6.2 g⋅mm	51°	779.8 g·mm²	321°		
10.1 g.mm	328°	5.1 g·mm	328°	6.2 g·mm	51°	3.1 g·mm	51°		
LEFT RIGHT		LEFT		RIGHT					
AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE	AMOUNT	ANGLE		
10.1 g·mm	328°	0.0 g⋅mm	0°	6.2 g·mm	51°	0.0 g·mm	0°		

3) Applying the right plane correction in the right plane, the corrections and residual unbalances are:

Again, the right plane unbalance is reduced to zero as it should be for a perfect correction. The residual force unbalance is slightly lower than that of example 2. This is due to the greater distance between planes. As the left plane moves to the left, the right plane correction becomes more like a force correction. The new left plane unbalance reflects all of the unbalance remaining in the rotor. The left plane unbalance could be used to predict the forces imposed on or transmitted through the mounting point in the left plane.

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