



## Unbalance Tolerance in Vector Space



**Joel M. Book**  
**John Haidler**

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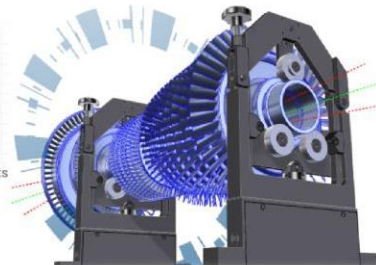
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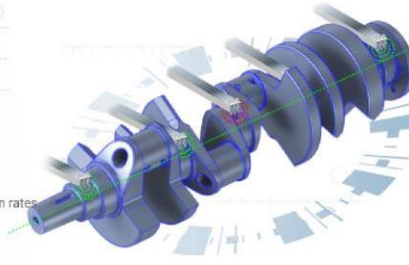
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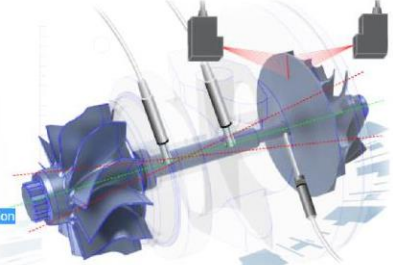
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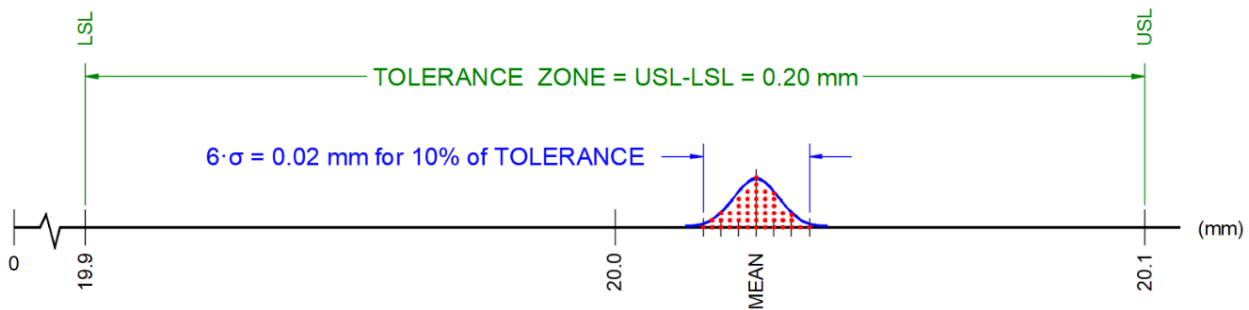
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## UNBALANCE TOLERANCE IN VECTOR SPACE

Joel M. Book, John Haidler

In general, most explanations of measurement tolerance are presented in one-dimensional terms. A typical example might be a shaft with a specified diameter of  $\varnothing 20$  mm and a tolerance of  $\pm 0.10$  mm. This is a one-dimensional measurement, with a bilateral tolerance. For a measure system to demonstrate a 10% GR&R (gage repeatability & reproducibility) the study variation ( $6\sigma$ ) or spread of data must not exceed 0.02 mm. Shown on a number line, it would look something like this:



In contrast to the diameter of a shaft, the unbalance of a rotor is a vector quantity, always having both magnitude and direction and possibly having any orientation relative to the rotor's angle reference (i.e., having positive or negative values of its cartesian-coordinate components). An unbalance tolerance should be understood in a vector sense. A simple Cartesian (X, Y) or polar (R,  $\Theta$ ) plot can be used to illustrate and better understand unbalance.

While actual unbalance is a vector quantity, most specifications of unbalance tolerance are expressed in one-dimensional forms, being only concerned with the magnitude of residual unbalance. Typical examples include:

- 1) Maximum unbalance of T g·mm;
- 2) Unbalance of  $U \pm T$  g·mm;

Further, balancer performance is also usually evaluated and qualified using one-dimensional methods, looking only at the magnitude of the measured unbalance. While this can be valid where only the final magnitude is of concern for the balancing process, the actual size of the unbalance tolerance zone may not be immediately obvious from reading the unbalance specification.

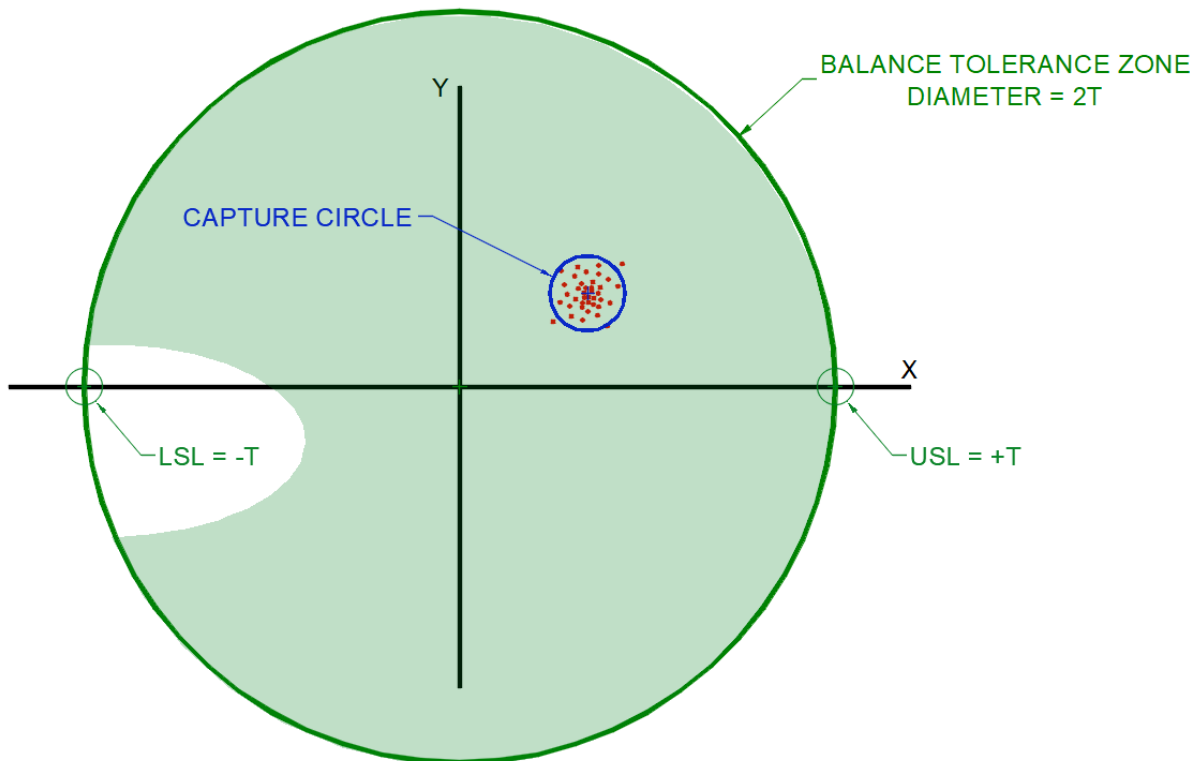
On rare occasions, two-dimensional balance requirements will be specified, as in:

- 3) Unbalance of  $U \pm T$  g·mm, within  $G^\circ$  of a specified axis, A;
- 4) Residual unbalance not to exceed T g·mm with a bobweight of weight W g at a given location (e.g. on a crankpin or at a mounting hole).

This document provides graphical explanations of how to interpret each of these cases in vector space. While this document presents only a single unbalance plane, as would be typical of a force-only unbalance tolerance, this presentation can be generalized to multiple planes or to force-couple specifications.

### 1) Maximum Unbalance of T g·mm.

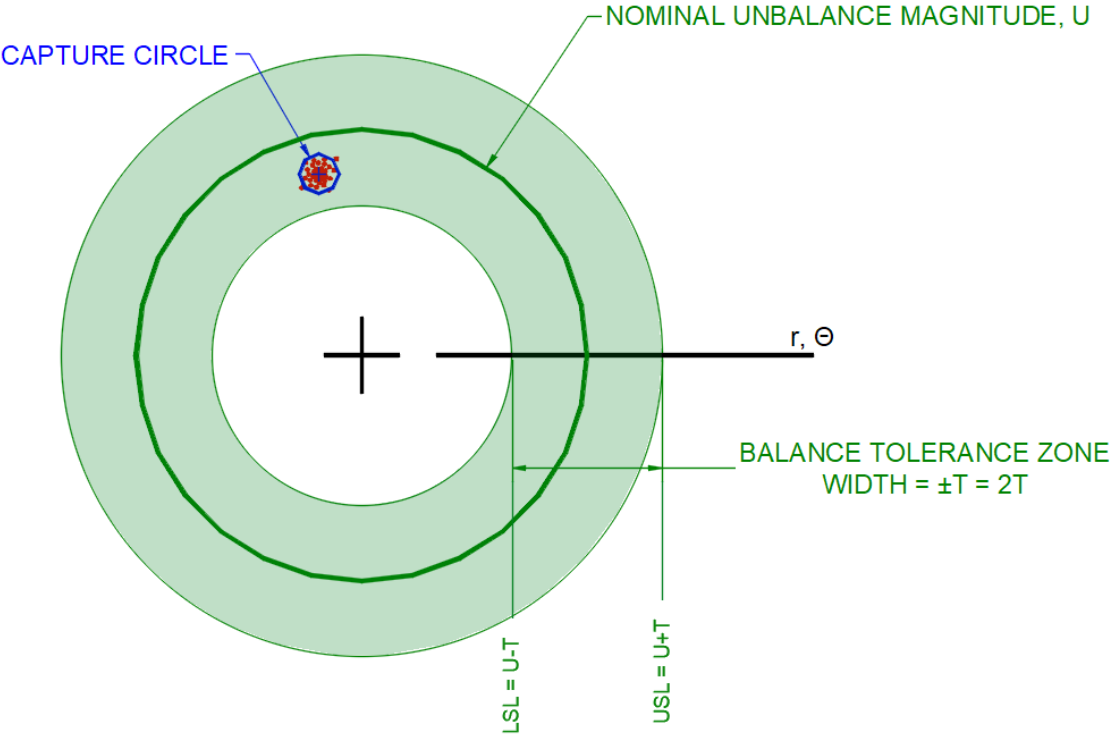
In this case, the nominal target unbalance is zero, and the tolerance zone is a circle of radius T g·mm when seen in a vector plane. This may also be expressed in Cartesian form as  $0 \pm T$  g·mm along any axis through the origin, allowing one to define upper and lower specification limits. The Upper Specification Limit, USL is +T g·mm, and the Lower Specification Limit, LSL is -T g·mm (along any axis that passes through the origin). This produces a total tolerance zone with width (diameter) of 2T g·mm.



This is equivalent to specification of maximum radius only in polar coordinates ( $R, \Theta$ ). Distributions are typically normal in magnitude ( $X, Y$  or  $R$ ) when sufficiently far from zero and non-normal when close to zero.

2) Unbalance of  $U \pm T$  g·mm.

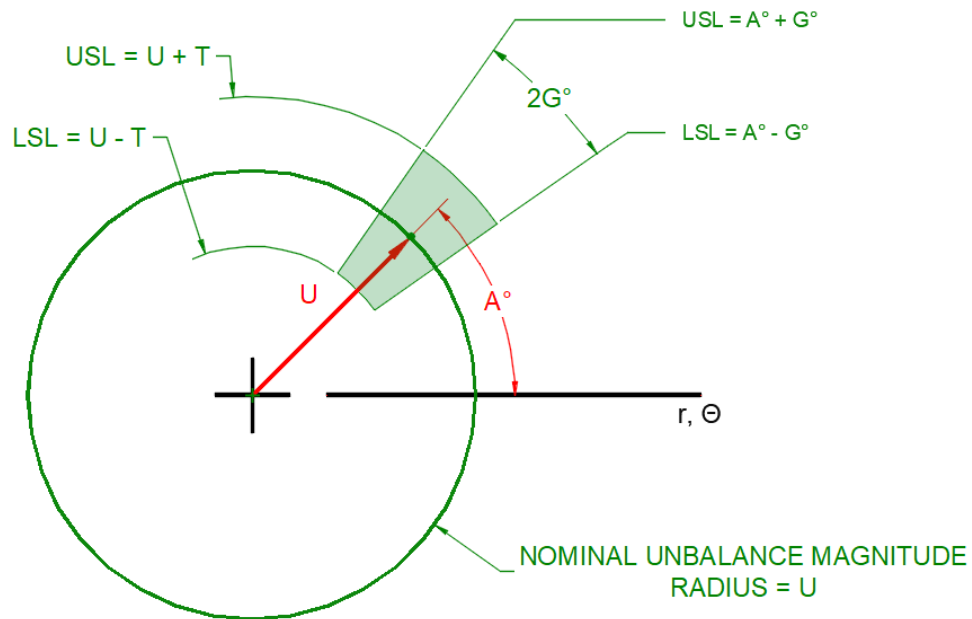
In this case, the nominal target unbalance is  $U$  g·mm, without an angle specification and with an unbalance tolerance of  $\pm T$  g·mm. This defines an annular (ring-shaped) tolerance zone for acceptable unbalance vectors. In polar coordinates, the USL along any line from the origin is  $U + T$  g·mm, while the LSL is  $U - T$  g·mm. The total width on a magnitude basis of the tolerance zone is  $2T$  g·mm, centered around the nominal value of  $U$  g·mm.



This is the form of tolerance zone that most one-dimensional gage repeatability, process capability, and GR&R methods assume. If  $U - T$  is sufficiently far from 0, the distribution is likely to be normal in magnitude only and 1D evaluation methods (Xbar & R, ANOVA) are valid measures provided the results are viewed with respect to the entire tolerance zone ( $2T$ ).

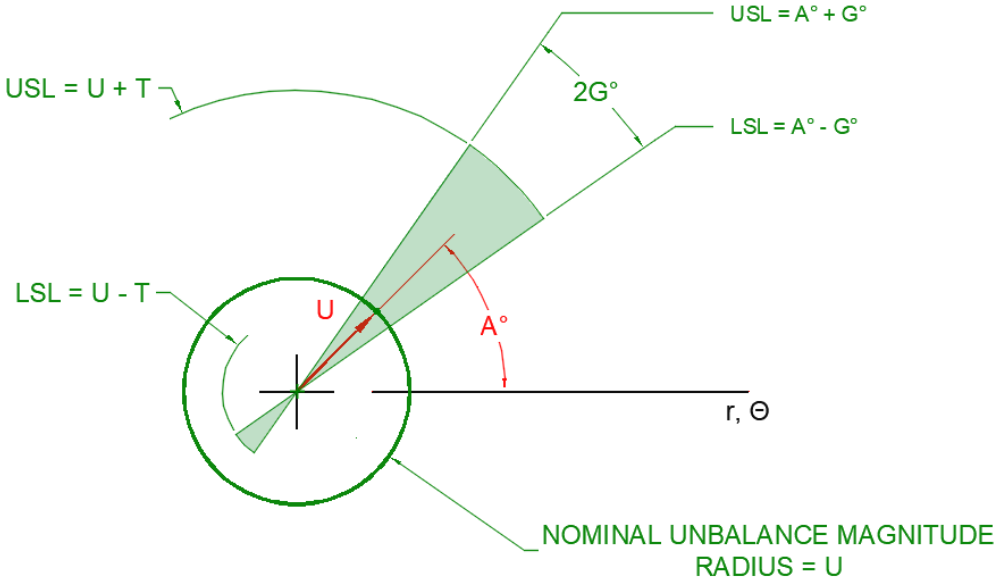
### 3) Unbalance of $U \pm T$ g·mm, at a defined angle.

In this case, the nominal target unbalance is  $U$  g·mm located exactly on an axis  $A$ . For the case with  $U$  greater than  $T$  the tolerance zone is a segment of an annulus, with USL and LSL in the unbalance amount direction of  $U + T$  and  $U - T$  g·mm respectively, and USL and LSL in the angle direction of  $\pm G^\circ$  on either side of axis  $A$ . The total width of the tolerance zone is  $2T$  g·mm in the amount direction and  $2G^\circ$  in the angular direction.



Caution should be exercised when evaluating angle in degrees as the range for patterns that include  $0^\circ$  can be easily mis-represented. Additional ambiguity can result from the angle convention used,  $0^\circ - 360^\circ$  or  $\pm 180^\circ$ . The  $2G^\circ$  value can also be determined in balance units, however the allowed range varies with the radius (magnitude) of the measurement, *i.e.* angular width in balance units at USL is greater than the angular width in balance units at LSL.

If T is larger than U, as may occasionally occur, then the tolerance zone is more complex. This results in a pair of unequal circular sectors on opposite sides of the zero point.



This is difficult to explain and likely to be a specification error.

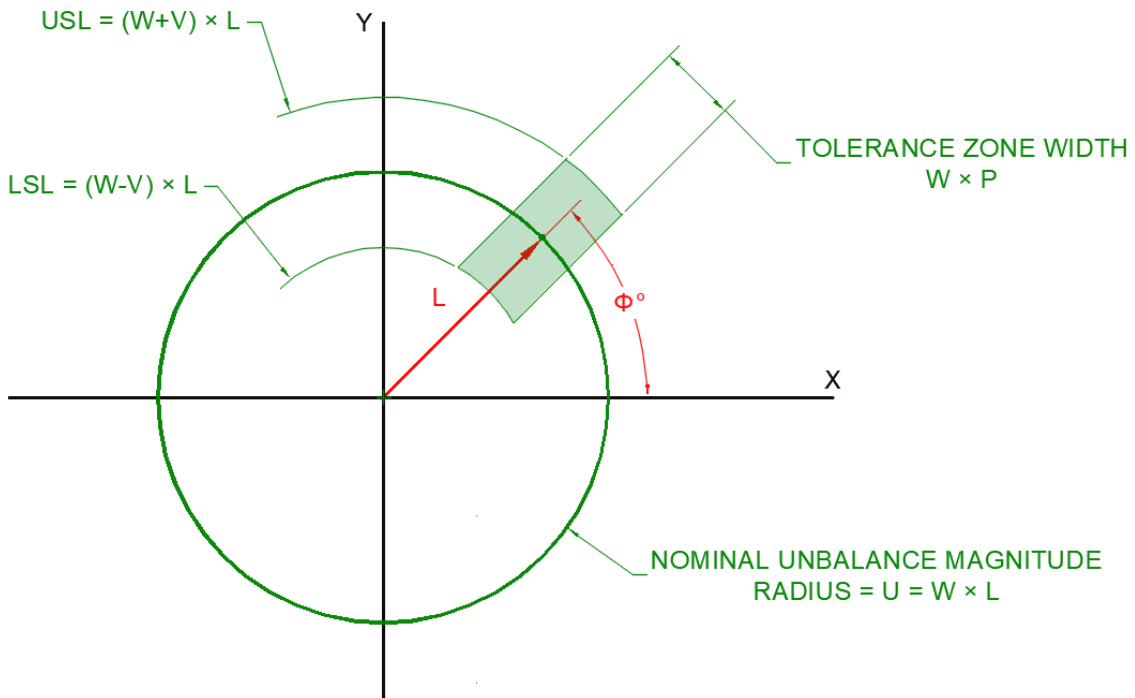


4) Maximum Unbalance of U g·mm with a BobWeight.

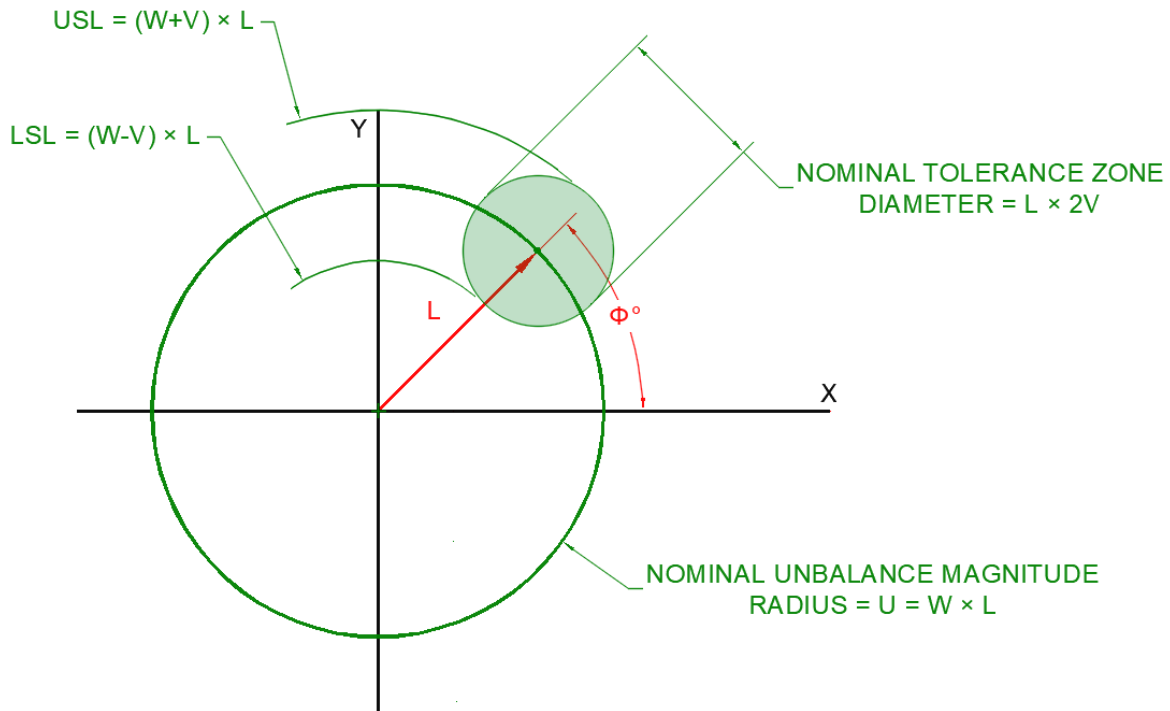
Residual unbalance can be specified in the same manner as case 1 with the addition of a bobweight. This is commonly expressed as residual unbalance not to exceed U g·mm with a bobweight of weight W g at a given location (L, φ). When a bobweight of the correct weight is actually used, the tolerance zone is exactly the same as described for case 1. Any variation in bobweight weight or location are immaterial provided they are within their respective design limits.

It is seldom practical to affix a physical bobweight to every part being balanced for mass production. Instead, the unbalance measurement may be performed without a bobweight and a compensation adjustment will be made in the balancer software to produce a part with the desired net outgoing unbalance.

Alternatively, a specification may require zero unbalance with a bobweight ( $W \pm V$  g) at a location (L, radius) and at an angle (Φ) with a positional tolerance (P). In this case the nominal residual unbalance target is equal to  $W \times L$  g·mm at the defined angle, φ. The unbalance tolerance zone is then defined by a function of W, V, L and P. There are several ways to do this. If P is small compared to L, it is convenient to ignore P in the tolerance zone for unbalance magnitude. If V is small compared to W, it is convenient to ignore V in the tolerance zone for the angular direction.



Even this simplified form can be a difficult tolerance zone to evaluate against in practice. It is usually more convenient to assume a circular tolerance zone with a radius equal to  $L \times V$  centered around the nominal position.



With this assumed circular tolerance, the USL is  $L \times (W + V)$  g·mm, and the LSL is  $L \times (W - V)$  g·mm, and the total tolerance zone width is  $L \times 2V$  g·mm.